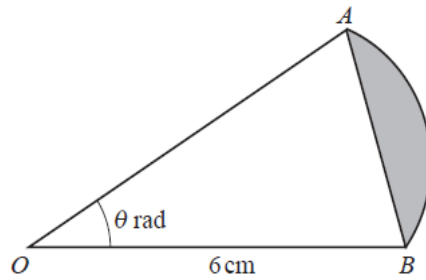


**A Level Mathematics A**  
**H240/01 Pure Mathematics**

**Question Set 4**

- 1** The function  $f$  is defined by  $f(x) = (x - 3)^2 - 17$  for  $x \geq k$ , where  $k$  is a constant.
- (a) Given that  $f^{-1}(x)$  exists, state the least possible value of  $k$ . [1]
- (b) Evaluate  $ff(5)$ . [2]
- (c) Solve the equation  $f(x) = x$ . [3]
- (d) Explain why your solution to part (c) is also the solution to the equation  $f(x) = f^{-1}(x)$ . [1]
- 2** Sam starts a job with an annual salary of £16 000. It is promised that the salary will go up by the same amount every year. In the second year Sam is paid £17 200.
- (a) Find Sam's salary in the tenth year. [2]
- (b) Find the number of complete years needed for Sam's **total** salary to first exceed £500 000. [4]
- (c) Comment on how realistic this model may be in the long term. [1]
- 3** Let  $f(x) = 2x^3 + 3x$ . Use differentiation from first principles to show that  $f'(x) = 6x^2 + 3$ . [6]
- 4** A cylindrical tank is initially full of water. There is a small hole at the base of the tank out of which the water leaks.
- The height of water in the tank is  $x$  m at time  $t$  seconds. The rate of change of the height of water may be modelled by the assumption that it is proportional to the square root of the height of water.
- When  $t = 100$ ,  $x = 0.64$  and, at this instant, the height is decreasing at a rate of  $0.0032 \text{ ms}^{-1}$ .
- (a) Show that  $\frac{dx}{dt} = -0.004\sqrt{x}$ . [2]
- (b) Find an expression for  $x$  in terms of  $t$ . [4]
- (c) Hence determine at what time, according to this model, the tank will be empty. [2]

5



The diagram shows a sector  $AOB$  of a circle with centre  $O$  and radius 6 cm.  
The angle  $AOB$  is  $\theta$  radians.  
The area of the segment bounded by the chord  $AB$  and the arc  $AB$  is  $7.2 \text{ cm}^2$ .

(a) Show that  $\theta = 0.4 + \sin \theta$ . [3]

(b) Let  $F(\theta) = 0.4 + \sin \theta$ .

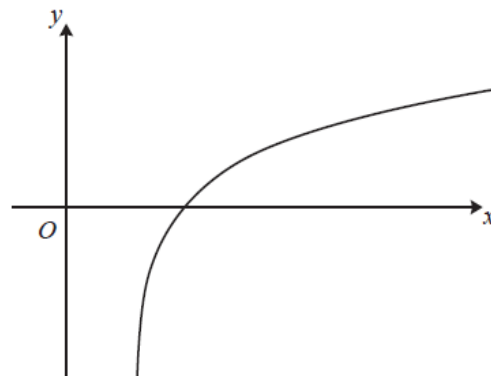
By considering the value of  $F'(\theta)$  where  $\theta = 1.2$ , explain why using an iterative method based on the equation in part (a) will converge to the root, assuming that 1.2 is sufficiently close to the root. [2]

(c) Use the iterative formula  $\theta_{n+1} = 0.4 + \sin \theta_n$  with a starting value of 1.2 to find the value of  $\theta$  correct to 4 significant figures.

You should show the result of each iteration. [3]

(d) Use a change of sign method to show that the value of  $\theta$  found in part (c) is correct to 4 significant figures. [3]

6



The diagram shows part of the curve  $y = \ln(x-4)$ .

(a) Use integration by parts to show that  $\int \ln(x-4) dx = (x-4) \ln|x-4| - x + c$ . [5]

(b) State the equation of the vertical asymptote to the curve  $y = \ln(x-4)$ . [1]

(c) Find the total area enclosed by the curve  $y = \ln(x-4)$ , the  $x$ -axis and the lines  $x = 4.5$  and  $x = 7$ . Give your answer in the form  $a \ln 3 + b \ln 2 + c$  where  $a$ ,  $b$  and  $c$  are constants to be found. [4]

**Total Marks for Question Set 4: 49 Marks**

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